E_m = fraction extracted
k = dispersed phase mass transfer coefficient
N_{Pe} = Peclet number, $\frac{1}{2}\frac{\mu_C}{\mu_C+\mu_D}\frac{ua}{D}$
N_{Sh} = Sherwood number, $2ka/D$
r = radial position in Handlos-Baron model
t = contact time
u = drop velocity
τ = dimensionless time, Dt/a^2
η = dimensionless position, $r/(a/2)$
μ = viscosity of drop liquid

Subscripts

c = continuous phase
D = dispersed phase
HB = based on Handlos-Baron solution

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Application of Integral Momentum Methods to Viscoelastic Fluids: Flow About Submerged Objects

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Viscoelastic flow problems involving the drag resistance of dilute polymer solutions have in recent years become of great interest to the petroleum industry and in naval applications (*4, 8, 12, 16, 20*). The rapid laminar flow of slightly viscoelastic fluids about submerged objects was first studied by Rajeswari and Rathna (*19*), who formulated the two-dimensional boundary-layer equations and looked in particular at the problem of flow near a stagnation point. Rajeswari and Rathna represented the stress field with the Coleman-Noll second-order fluid (*9, 15, 26*). Essentially the same problem was reanalyzed by Beard and Walters (*2*) in a somewhat different fashion. A different attack on viscoelastic boundary-layer theory has been made by White and Metzner (*27, 28*), who noting that the second-order fluid concept is inadequate at the high deformation rates involved in the boundary-layer problem, developed a more complex constitutive equation which introduces non-Newtonian viscosity as well as second-order viscoelastic effects. They go on to indicate the range of validity of viscoelastic boundary-layer theory and relate it specifically to the theory of purely viscous non-Newtonian boundary layers developed by Schowalter (*22*) and by Acrivos, Shah, and Petersen (*1, 23*) (see also reference *11*). Essentially it was found that these purely viscous solutions are valid up to rather high values of the Weissenberg number* based upon the boundary-layer thickness, but to only small values of Weissenberg numbers based upon the distance along the surface from the forward stagnation point. High boundary-layer thickness Weissenberg numbers imply that highly elastic behavior is required to produce any measurable effects in this flow field. In the range of such large Weissenberg numbers, White and Metzner found several external velocity fields which would allow transformation of the boundary-layer equation to an ordinary differential equation, but the problem of greatest interest, the constant mainstream velocity, cannot be so transformed. Recently, Denn (*10*) generalized the work of White and Metzner by using a more complex constitutive law and obtained additional transformation solutions. However Denn was not able to obtain a transformation for the constant mainstream velocity case.

There exists an alternative approach to this problem in which the equations of motion are integrated in a special manner. This integral momentum method for Newtonian fluids is reviewed by Schlichting (*21*); it was introduced into non-Newtonian fluid dynamics by Bogue (*5*) and by Acrivos, Shah, and Petersen (*1*). The flow of purely viscous non-Newtonian fluids about submerged objects has been studied by this procedure by Acrivos, Shah, and Petersen (*1, 23*). Integral momentum procedures were introduced into viscoelastic fluid dynamics by Metzner and White (*17*) in order to study the entrance region. Their use of a modified von Karman method suggests a unique approach to many viscoelastic flow problems, for it enables one to obtain approximate solutions without presuming a particular constitutive equation for the stress. Rather, one may directly use viscometric laminar shear flow relationships between the shear and normal stress components and the shear rate. The saving in effort by this method is considerable; one need only read the papers of White and Metzner (*27*) and Denn (*10*) to see this. In this communication, we will apply this perhaps unique procedure to the flow past a semi-infinite flat plate.

EQUATIONS OF MOTION

Consider the rapid flow of an infinite viscoelastic fluid with a mainstream velocity $U(x)$ past a submerged flat plate, which stretches from the origin along the x axis. The fluid adheres to the flat plate and its velocity varies rapidly through a distance $\delta(x)$ to the mainstream value. The equations of motion may be simplified by the Prandtl kinematic approximations:

$$u \sim U, \quad v \sim U\,\delta/x, \quad \frac{\partial u}{\partial y} \sim \frac{U}{\delta}, \quad \text{etc.}$$

This allows us to rewrite the equations of motion for values of y less than $\delta(x)$ as [for Newtonian fluids see Schlichting (*21*) and for viscoelastic fluids see White and Metzner (*27*) and Rajeswari and Rathna (19)]:

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \frac{\partial t_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \quad (1a)$$

* The Weissenberg number, named for Karl Weissenberg, who was one of the pioneers in nonlinear viscoelastic hydrodynamics, represents a ratio of viscoelastic to viscous forces in a flow field. Applications of this dimensionless group to different theoretical and industrial problems are discussed in the literature (*26* to *28, 17, 4, 18, 10*).

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$$0\left(\frac{U^2}{x^2}\right) = -\frac{\partial p}{\partial y} + \frac{\partial t_{yy}}{\partial y} \qquad (1b)$$

where we have neglected the x variation of τ_{xy}. The pressure field throughout the infinite fluid may be rewritten as

$$p(x, y) = t_{yy}(x, y) - \tau_{yy}(x, 0) \qquad (2)$$

Substitution of this expression for the pressure field into Equation (1a) yields

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = +\frac{d\tau_{yy}(x, 0)}{dx} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial}{\partial x}(t_{xx} - t_{yy}) \qquad (3)$$

As y approaches the value δ, Equation (3) may be expressed as

$$\rho U\frac{dU}{dx} = +\frac{d\tau_{yy}}{dx}(x, 0) + \left[\frac{d(t_{xx} - t_{yy})}{dx}\right]_{y=\delta} \qquad (4)$$

This allows us to rewrite Equation (3) as

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - U\frac{dU}{dx}\right] = -\left[\frac{d(t_{xx} - t_{yy})}{dx}\right]_{y=\delta} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial}{\partial x}(t_{xx} - t_{yy}) \qquad (5)$$

We will now integrate Equation (5) with respect to y from the rigid flat plate at y equal to zero to some value h everywhere greater than the boundary-layer thickness. In so doing we will neglect the existence of significant normal stresses in the mainstream. Integrating in the manner detailed by Schlichting (*21*), we obtain

$$\int_o^h \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - U\frac{dU}{dx}\right]dy = \int_o^h\left[\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial}{\partial x}(t_{xx} - t_{yy})\right]dy \qquad (6)$$

and

$$\rho\frac{d}{dx}\int_o^h u(U - u)dy + \rho\left(\frac{dU}{dx}\right)\int_o^h (U - u)dy = \tau_w - \frac{d}{dx}\int_o^h (t_{xx} - t_{yy})dy \qquad (7)$$

We may replace h by δ in Equation (7). The above equation represents the generalization of the von Karman integral momentum equation to *slightly viscoelastic* non-Newtonian systems. The integral momentum equations of Acrivos, Shah, and Petersen represent the special case of this equation applicable to the three-dimensional power law fluid (*3, 11, 22*). The term *slightly viscoelastic* is used only in the sense that the presence of viscoelasticity does not perturb the velocity field to such an extent as to make the boundary-layer approximations unavailable in simplifying the equations of motion. We shall use this phrase in a more meaningful sense in the next section.

Our equation of motion may be integrated in an alternative fashion due to Wieghardt (*29, 21*). Rather than integrating directly we first multiply through by the x component of the velocity vector u to obtain the mechanical energy equation and at this point one integrates between y equal to zero and y equal to h. In such a manner, one obtains

$$\frac{d}{dx}\left\{\frac{\rho}{2}\int_o^\delta u(U^2 - u^2)dy + \int_o^\delta u[(t_{xx} - t_{yy})]dy\right\} = \int_o^\delta\left[\tau_{xy}\frac{\partial u}{\partial y} + (t_{xx} - t_{yy})\frac{\partial u}{\partial x}\right]dy \qquad (8)$$

FLAT PLATE PROBLEM

In order to proceed further, the dependence of U upon x must be specified and we shall consider here the most important case, namely, that in which U is taken to be independent of position along the plate. For this case, Equation (1) may be written as

$$\rho\frac{d}{dx}\int_o^\delta u(U - u)dy = \tau_w - \frac{d}{dx}\left[\int_o^\delta (t_{xx} - t_{yy})dy\right] \qquad (9)$$

or

$$\rho U^2\frac{d}{dx}(\theta\,\delta) = \tau_w - \frac{d}{dx}\left[\int_o^\delta (t_{xx} - t_{yy})dy\right] \qquad (10)$$

in which

$$\theta = \int_o^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) \qquad (11)$$

At this point in our analysis, we must make a basic assumption. We conjecture that the velocity field for y less than δ may be written in the form

$$u = Uf'\ (y/\delta \text{ only}) \qquad (12)$$

$$f'(0) = 0 \qquad f'(1) = 1$$

Combining Equations (12) and (11), one obtains by inspection that θ is independent of x. Therefore Equation (10) yields

$$\rho U^2 \theta\frac{d\delta}{dx} = \tau_w - \frac{d}{dx}\left[\int_o^\delta (t_{xx} - t_{yy})dy\right] \qquad (13)$$

It now remains to relate the components of the stress field to the velocity field and thus to the boundary-layer thickness δ. One may evaluate the wall shearing stress τ_w by noting that at y equal to zero

$$u = v = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial y} \sim \frac{U}{\delta} \qquad \frac{\partial v}{\partial y} = 0$$

It can thus be seen that any stress component at the surface of the plate is a unique function of the instantaneous velocity gradient $\partial u/\partial y$ or better a functional of the history of this gradient. As may be seen from the recent work of Metzner, White, and Denn (*18*) the stresses near the origin of the plate are determined by the local shear *strain* rather than the velocity gradient. When one proceeds along the plate for such a distance that the residence times of the fluid particles are of the same order as the fluid's natural time, the stresses are then given by the instantaneous velocity gradient. We again invoke the restriction that our analysis only applies to *slightly viscoelastic* fluids; however here we explicitly presume that the natural time of the fluids considered is significantly less than the residence time of particles of these fluids moving through the boundary layer parallel to the flat plate. Thus the stress is only determined by the strain in the immediate neighborhood of the stagnation point. While this region may possess velocity profiles somewhat different from those observed in purely viscous fluids, we do not think that there will be a significant influence further downstream. In any case our analysis would seem to be a good approximation for dilute polymer solutions with natural times of order a second or less, but it would be unreasonable for polymer melts with natural times of the order of minutes. Bearing in mind the above discussion, we express the wall shear stress as

$$\tau_w = \mu \left[\left(\frac{\partial u}{\partial y} \right)_w^2 \right] \left(\frac{\partial u}{\partial y} \right)_w = \mu_w \, [f''(0)] \frac{U}{\delta} \quad (14)$$

The boundary-layer normal stresses present a somewhat difficult problem as they must be evaluated and integrated across the boundary-layer thickness. To obtain these stresses we must analyze an acceleration tensor expansion. [The basis and application of acceleration tensors in nonlinear viscoelasticity was devised by Rivlin and his colleagues in the 1950's. Pertinent discussions are found in our earlier papers (*26, 27, 18*) and in the monograph by Fredrickson (*11*).] If one could presume that flow within the boundary layer was the same as laminar shearing motions in a viscometer, then all acceleration tensors higher than the second will be zero and the normal stresses may be determined from rheogoniometric (*6, 13, 15*) and capillary thrust (*24, 25*) data. This is not exactly the case and there will be a contribution from the third acceleration tensor of the form

$$u \frac{\partial}{\partial x} \left[\left(\frac{\partial u}{\partial y} \right)^2 \right] + v \frac{\partial}{\partial y} \left[\left(\frac{\partial u}{\partial y} \right)^2 \right] + 2 \frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial y} \right)^2$$

to the normal stress difference. In this note we shall however neglect this contribution and we will express the normal stress difference as

$$(t_{xx} - t_{yy}) = (\beta_1 - \beta_2) \left(\frac{\partial u}{\partial y} \right)^2 = (\beta_1 - \beta_2) \, [f'']^2 \left(\frac{U}{\delta} \right)^2 \quad (15)$$

As noted above, both sets of rheological parameters μ_w and $(\beta_1 - \beta_2)$ are functions of the velocity gradient $\partial u / \partial y$ and may be determined from laminar shearing motion data. On the basis of this experimental data, we may well approximate both rheological properties by power laws. Thus we write

$$\mu_w = K \left(\frac{\partial u}{\partial y} \right)_w^{n-1} = K \, [f''(0)]^{n-1} \left(\frac{U}{\delta} \right)^{n-1} \quad (16a)$$

$$(\beta_1 - \beta_2) = m \left(\frac{\partial u}{\partial y} \right)^{s-2} = m \left[f'' \left(\frac{y}{\delta} \right) \right]^{s-2} \left(\frac{U}{\delta} \right)^{s-2} \quad (16b)$$

Table 1 summarizes existing data on the shear rate dependence of normal and shearing stresses of hydrocarbon polymers in hydrocarbon solvents. It is well known that presuming the exponent n in Equation (16*a*) to be a constant is a good approximation over a wide shear rate range. Inspection of the data, especially that of Philippoff (*6*) and of Ginn and Metzner (*13*), indicates that s is reasonably constant also. Substitution of Equations (14), (15), and (16) into Equation (13) and rearrangement yield

$$[\rho \, U^2 \, \theta \, \delta^n - (s-1) \, m \, U^s \, F \, \delta^{n-s}] d\delta = K [f''(0)]^n U^n \, dx \quad (17)$$

in which

$$F = \int_0^1 \left[f'' \left(\frac{y}{\delta} \right) \right]^s d \left(\frac{y}{\delta} \right) \quad (18)$$

denotes a constant. After integration with the boundary condition

$$\delta(0) = 0 \quad (19)$$

Equation (17) will give the variation of the thickness δ along the plate.

TABLE 1. RHEOLOGICAL CHARACTERISTICS OF FLUIDS

Shear rate range	n	s	$s - n$	Ref.
Very low				
$0(1)$ sec.$^{-1}$	1	2	1	(*6, 15*)
Intermediate	<1	<2	<1	(*6, 13, 24*)
$0(50$ to $500)$ sec.$^{-1}$		$\sim 2n$		
High				
$0(50{,}000)$ sec.$^{-1}$	$n \to 1$	$s \to 1$	$<<1$	(*6, 24, 25, 13*)

Two points are immediately evident from Equation (17). First, if the value of the normal stress index s in Equation (17) is unity then viscoelasticity will have no effect upon the boundary-layer thickness. On the other hand if

$$s - n \geqq 1 \quad (20)$$

then it will be impossible to satisfy Equation (19). Values of $(s - n)$ of this magnitude represent just the properties presumed in the boundary-layer analyses of Rajeswari and Rathna (*19*), Beard and Walters (*2*), and of White and Metzner (*27, 28*), though not by Denn (*10*).

From the summary of data in Table 1, it may be seen that only at very low levels of shear rate, levels of no interest in boundary-layer analyses, does the difference $(s - n)$ appear to approach unity.

The experimental data cited in Table 1 also point out that s approaches unity at high rates of shear. We thus have the very interesting effect of normal stresses continuing to increase with rate of mainstream flow but the deviations from the purely viscous boundary-layer theory decrease and eventually disappear. The source of this peculiarity may be found by analyzing Equations (13), (15), (16), and (17), for if the stress difference $(t_{xx} - t_{yy})$ varies linearly with the first power of (U/δ), then

$$\frac{d}{dx} \left[\int_0^\delta (t_{xx} - t_{yy}) dy \right] = \frac{d}{dx} [m \, F \, U] = 0 \quad (21)$$

That is, a value of s equal to unity is just the condition under which the viscoelastic normal force, integrated throughout the boundary layer, is independent of distance along the plate and hence affects neither the velocity nor the frictional drag.

Equation (17) may be integrated to yield

$$\left(\frac{\delta}{x} \right)^{n+1} \left[1 - \frac{(n+1)(s-1)}{n-s+1} \frac{F}{\theta} \left(\frac{x}{\delta} \right)^s \frac{N_{Ws}}{N_{Re}} \right] = \frac{A(n+1)}{\theta \, N_{Re}} \quad (22)$$

in which

$$A = [f''(o)]^n \quad (23)$$

and the dimensionless Reynolds and Weissenberg numbers for the power law behavior presumed are given by

$$N_{Re} = \frac{x^n \, U^{2-n} \, \rho}{K} \qquad N_{Ws} = \frac{m}{K} \left(\frac{U}{x} \right)^{s-n} \quad (24)$$

Expanding about the purely viscous solution in terms of the Weissenberg number allows us to solve for the boundary-layer thickness:

$$\left(\frac{\delta}{x} \right) = \left(\frac{\delta}{x} \right)_o \left[1 + \frac{(s-1)F}{\theta(n+1-s)} \frac{N_{Ws}}{N_{Re}} \left(\frac{\delta}{x} \right)_o^{-s} + \dots \right] \quad (25)$$

TABLE 2. VALUES OF COEFFICIENTS IN EQUATION (28)

n	s $(2n)$	a	b
1.0	2.0	—	—
0.8	1.6	270	1.0
0.7	1.4	76.5	1.0
0.6	1.2	15.7	1.0
0.5	1.0	0	1.0

where

$$\left(\frac{\delta}{x}\right)_o = \left[\frac{A(n+1)}{\theta N_{Re}}\right]^{\frac{1}{n+1}} \quad (26)$$

The local drag coefficient may be defined as

$$c_{f,x} = \frac{2\tau_w}{\rho U^2} = \frac{2KA}{\rho U^2}\left(\frac{U}{\delta}\right)^n = \frac{2A}{N_{Re}}\left(\frac{x}{\delta}\right)^n \quad (27)$$

and from Equations (25) and (26)

$$c_{f,x} = [c_{f,x}]_o \left[1 - a N_{Ws} N_{Re}{}^b + o\,(N_{Ws}{}^2)\right] \quad (28)$$

where

$$a = \frac{2n(s-1)}{(n+1-s)}\,\frac{A}{\theta}\left[\frac{F(n+1)}{\theta}\right]^{s/n+1}; \qquad b = \frac{s+1-n}{n+1} \quad (29)$$

$$[c_{f,x}]_o = 2F\left[\frac{\theta}{F(n+1)}\right]^{\frac{n}{n+1}} N_{Re}{}^{-\frac{1}{n+1}} \quad (30)$$

When s is greater than unity, viscoelasticity tends to increase the boundary-layer thickness and decrease the frictional drag. Typical values of a and b are given in Table 2 where we have chosen s to be $2n$, evaluated F with a linear profile and obtained θ from Acrivos, Shah, and Petersen (*1*).

It is of interest to compare the results of this investigation with other work in the literature. Rajeswari and Rathna (*19*) and Beard and Walters (*2*) have given approximate solutions for flow of a second-order fluid into a stagnation point. Both find that viscoelasticity decreases the boundary-layer thickness and increases drag. Denn (*10*) finds that δ and $c_{f,x}$ may increase or decrease depending upon geometry and the exponents s and n. For the geometry considered here, a perturbation solution given by Denn is in qualitative agreement with the above result. It is also of interest to note that the effect of viscoelasticity in creeping flow is to reduce drag (*7, 14*).

ACKNOWLEDGMENT

This work has been discussed with Professors A. B. Metzner and M. M. Denn of the University of Delaware whose comments are appreciated. The contents of this paper are from the author's Ph.D. thesis (1965), which was supported during the course of the work herein (1963) by the Office of Naval Research.

NOTATION

- A = dimensionless velocity gradient at plate, Equation (23)
- a = coefficient in Equation (28), defined in Equation (29)
- b = exponent in Equation (28), defined in Equation (29)
- $c_{f,x}$ = local friction factor (drag coefficient), Equation (27)
- f' = dimensionless velocity, u/U, Equation (12)
- F = integral defined by Equation (18)
- K = consistency index, Equation (16*a*)
- m = normal stress index, Equation (16*b*)
- n = flow behavior index, Equation (16*a*)
- N_{Re} = Reynolds number, Equation (24)
- N_{Ws} = Weissenberg number, Equation (24)
- o = order of magnitude
- p = pressure
- s = normal stress exponent, Equation (16*b*)
- t_{ij} = component of stress tensor expressed τ_{ij} (total stress) $= -p\,\delta_{ij} + t_{ij}$
- u, v = local velocities in the x and y coordinate directions
- U = mainstream velocity
- x, y = coordinate labels
- δ = boundary-layer thickness
- β_1, β_2 = normal stress coefficients, Equation (15)
- μ = viscosity in laminar shear flow, Equation (14)
- θ = momentum thickness divided by boundary-layer thickness, Equation (11)
- ρ = density
- τ_{xy}, τ_w = shearing stress

Subscripts

- w = flat plate (wall) conditions
- 0 = purely viscous

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